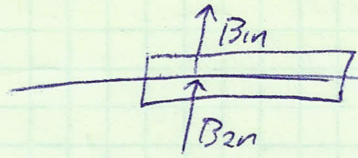
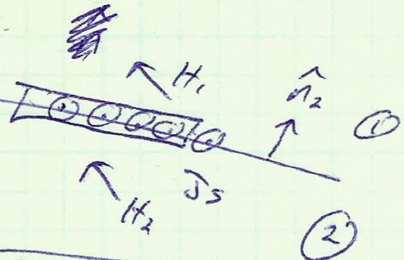
Magnetic Boundary Conditions

$$\oint_s \vec{B}\cdot d\vec{s} = 0 \Rightarrow \boxed{B_{1n} = B_{2n}}$$

$$\oint_c \vec{H}\cdot d\vec{l} = I$$





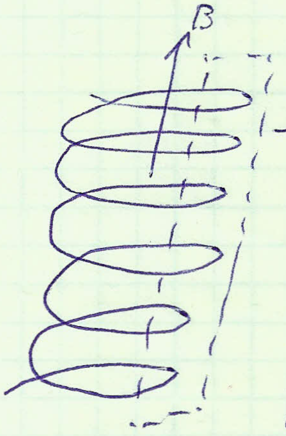
$$\boxed{\mu_1 H_{1n} = \mu_2 H_{2n}}$$

$$\boxed{\hat{n}_2 \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s}$$

curl selects tangential component

when $J_s = 0$, $\boxed{H_{1t} = H_{2t}}$

- Inductance



$$\oint \vec{H}\cdot d\vec{l} = I$$

$$Hl = NI$$

$$B = \frac{\mu N I}{l}$$

for long, thin solenoid

- Self and mutual inductance calculated by magnetic flux linkage

Magnetic flux $\Phi = \int_s \vec{B}\cdot d\vec{s}$

for one loop of the solenoid, $\Phi = \int_s \frac{\mu N I}{l} ds = \frac{\mu N I}{l} S$ ← surface area of loop

total flux linkage $\Lambda = N\Phi = \mu \frac{N^2}{l} I S$

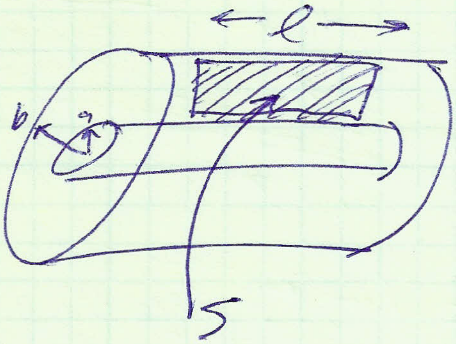
Self inductance $L = \frac{\Lambda}{I} = \mu N^2 \frac{S}{l}$ ← area, ← length

Unit of inductance is <u>Henrys</u>

or Webers/Ampere

To calculate inductance of transmission line:

find flux linkage through a surface between the two conductors



To understand why, consider a short circuit at the end

These areas become the area enclosed by the circuit

For the coaxial case, we previously found $\vec{B} = \hat{\phi}\dfrac{\mu I}{2\pi r}$

$$\Phi = \ell\int_a^b B\,dr = \ell\int_a^b \frac{\mu I}{2\pi r}\,dr = \frac{\mu I \ell}{2\pi}\ln\left(b/a\right)$$

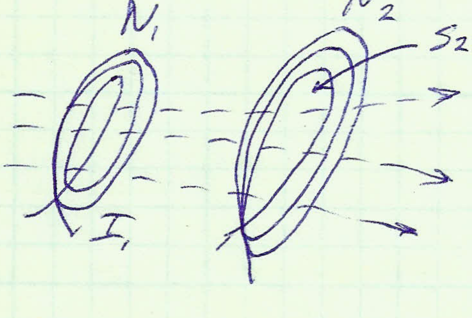
$$L' = \frac{L}{\ell} = \frac{\Phi}{I\ell} = \boxed{\frac{\mu}{2\pi}\ln\left(b/a\right)}$$

- Mutual Inductance

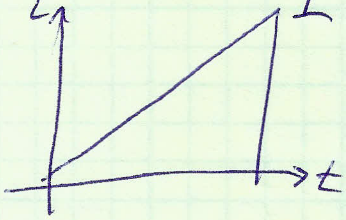
$$\Phi_{12} = \int_{S_2} \vec{B}_1 \cdot d\vec{s}$$

$$\Lambda_{12} = N_2 \Phi_{12}$$

$$\boxed{L_{12} = \frac{\Lambda_{12}}{I_1} = \frac{N_2}{I_1}\int_{S_2} \vec{B}_1 \cdot d\vec{s}}$$



Magnetic Energy



ramp up current from 0 to I

$$W_m = \underbrace{\int p\,dt}_{\text{power}} = \int i v\,dt = \underbrace{L \int_0^I i\,di}_{\text{from } v = L\,di/dt} = \boxed{\tfrac{1}{2} L I^2}$$

substitute $L = \mu N^2 S / \ell$

and $B = \mu N I / \ell$ ← rearrange to $I = \dfrac{B\ell}{\mu N}$

$$W_m = \tfrac{1}{2} L I^2 = \tfrac{1}{2}\left(\frac{\mu N^2 S}{\ell}\right)\left(\frac{B\ell}{\mu N}\right)^2$$

$$W_m = \frac{1}{2}\frac{B^2}{\mu}\ell S = \frac{1}{2}\mu H^2 V$$ ← volume

Magnetic energy density

$$\boxed{w_m = \frac{W_m}{V} = \frac{1}{2}\mu H^2}$$